

Enhancing Mathematical Skills of Grade 6 Learners Through Simple Word Problems Using PEMDAS: Basis for Development of Play-Based Instructional Material

Kenneth Valdez

Osias Educational Foundation

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Email of Corresponding Author: valdezken1234@gmail.com ; ORCID: [0009-0002-8577-0268](https://orcid.org/0009-0002-8577-0268)

Abstract

This study aimed to enhance the mathematical skills of Grade 6 learners through simple word problems using PEMDAS as the basis for developing a play-based instructional material titled Snake and Ladder Anchored to the Wizard. Anchored on constructivism, sociocultural, cognitive load, spiral curriculum, behaviorist theory, and the Concrete–Pictorial–Abstract (CPA) approach, the study used a quasi-experimental one-group pretest–posttest design. The respondents were 55 Grade 6 learners from Paratong Integrated School during School Year 2025–2026, selected through total enumeration. Data were gathered using a researcher-made PEMDAS achievement test, supported by learning modules, activity sheets, visual organizers, guided exercises, and the play-based instructional material. Mean, standard deviation, and paired sample t-test were used for data analysis. Results showed that learners’ mathematical performance improved from a low pretest mean score of 8.89 to a high posttest mean score of 15.02. The paired sample t-test revealed a significant difference between pretest and posttest scores, with a t-value of -14.64 and a p-value of <0.001. The findings indicate that the CPA-based PEMDAS instructional material improved learners’ conceptual understanding, procedural fluency, problem-solving accuracy, engagement, and confidence in solving multi-step word problems. The study concludes that structured, scaffolded, visual, and play-based instructional materials can strengthen foundational mathematics skills and guide classroom remediation. It recommends integrating CPA-based and PEMDAS-focused activities in mathematics instruction. The study supports SDG 4, SDG 5, and SDG 9 by promoting inclusive, equitable, and innovative mathematics learning through low-cost, localized, and adaptable instructional support for resource-constrained classrooms and similar educational settings nationwide.

Keywords: Concrete–Pictorial–Abstract Approach; Grade 6 Learners; Instructional Materials; Mathematical Performance; Order of Operations; PEMDAS; Play-Based Instructional Material; Snake and Ladder Anchored to the Wizard; Word Problems

1. Introduction

Mathematics is a foundational area of elementary education because it develops logical reasoning, computational accuracy, and problem-solving skills needed for higher-level learning and daily life applications. For Grade 6 learners, mastery of mathematical operations and the correct application of the order of operations through PEMDAS are essential in solving multi-step word problems. However, many learners continue to experience difficulty in applying operational hierarchy, particularly when problems involve parentheses, multiplication, division, addition, and subtraction. These difficulties affect learners’ accuracy, confidence, and motivation in mathematics. This study focuses on enhancing the mathematical skills of Grade 6 learners through simple word problems using PEMDAS as the basis for developing play-based instructional material, particularly the Snake and Ladder Anchored to the Wizard activity. It integrates the Concrete–Pictorial–Abstract (CPA) approach, guided practice, visual representations, scaffolding, and learner-centered activities to improve conceptual understanding, procedural fluency, and problem-solving confidence.

Mathematics serves as a bridge between logical reasoning, problem-solving, and practical application. In the elementary years, learners are expected to master fundamental operations that prepare them for more complex mathematical tasks. Grade 6 learners, in particular, are at a critical stage where they must move from simple computation to multi-step problem-solving requiring both conceptual understanding and procedural accuracy. Despite this expectation, learners often solve problems from left to right without applying the correct hierarchy of operations. Research by Magallanes and Ubalde (2023) revealed that learners commonly fail to apply multiplication and division before addition and subtraction and misunderstand the treatment of operations inside parentheses. These misconceptions persist when instruction focuses mainly on memorization rather than deeper conceptual understanding. Globally, numeracy remains a major concern. The United Nations Educational, Scientific, and Cultural Organization, UNESCO (2020), reported that many elementary learners experience difficulty in solving mathematical expressions involving multiple operations. Similarly, data from the

Program for International Student Assessment (PISA, 2022) show that learners in countries with strong mathematics performance, such as Singapore and South Korea, outperform their peers in the Philippines, partly due to innovative teaching strategies and well-developed instructional materials. In the Philippine context, the Department of Education (DepEd, 2022) linked low National Achievement Test performance to poor understanding of the order of operations. At Paratong Integrated School in Bangar, La Union, diagnostic results showed that many Grade 6 learners struggled with multiplication, division, subtraction, regrouping, and determining the correct sequence of operations. These challenges were further influenced by limited instructional materials, overcrowded classrooms, and insufficient individualized learning support.

The literature emphasizes that mathematical proficiency is essential for logical reasoning, critical thinking, and academic success. Magallanes and Ubalde (2023) noted that understanding the sequence of operations through PEMDAS helps learners solve multi-step word problems accurately and confidently. Strong mathematical skills also improve motivation, participation, and self-efficacy, while persistent difficulties weaken academic performance and problem-solving confidence (Radford University, 2020). Related studies show that guided exercises, visual aids, contextualized word problems, and scaffolded instructional materials help learners understand mathematical concepts more effectively than rote memorization. Visual representations, including step-by-step PEMDAS worksheets and diagrams, allow learners to understand the hierarchy of operations and apply procedures accurately (Hand et al., 2024). Multi-step word problems supported by guided instruction reduce misconceptions, improve computation accuracy, and strengthen problem-solving confidence (Magallanes and Ubalde, 2023; Radford University, 2020). The Concrete-Pictorial-Abstract (CPA) approach is also strongly supported in the literature. Powell and Fuchs (2021) emphasized that evidence-based mathematics practices, including CPA, improve mathematical performance by strengthening both conceptual and procedural skills. Bouck and Park (2020) found that CPA-based instruction produced meaningful gains in computation and problem-solving abilities. Carboneau, Marley, and Selig (2022) highlighted the role of manipulatives in improving mathematical achievement and conceptual understanding, while the National Council of Teachers of Mathematics (NCTM, 2023) reaffirmed the importance of multiple representations in mathematics instruction. CPA instruction, introduced by Bruner (1966), supports learning through concrete, pictorial, and abstract stages. Kurniawan et al. (2020) found that CPA instruction improved mathematical problem-solving skills among elementary learners. Mohamed and Meng (2021) reported that CPA improved algebraic understanding and reduced mathematics anxiety, while Yıldırım and Karabulut (2023) found substantial improvements in test scores after CPA implementation. These findings confirm the value of CPA as a structured approach for helping learners understand complex mathematical concepts. Structured instructional materials also support effective teaching. Hattie (2021) identified explicit instruction and worked examples as effective strategies for improving student achievement, particularly in hierarchical procedures such as PEMDAS. Rosenshine (2020) emphasized modeling, scaffolding, guided practice, and frequent review as essential principles for mastering complex skills. Hand et al. (2024) further demonstrated that visual and scaffolded problem-solving materials improved learners' accuracy and engagement in solving multi-step word problems.

At the global level, numeracy challenges remain a concern in basic education, particularly in relation to multi-step mathematical expressions and problem-solving tasks. UNESCO (2020) emphasized the need for inclusive and effective learning opportunities, while the Program for International Student Assessment (PISA, 2022) reflected continuing differences in mathematics performance across countries. Nationally, DepEd (2022) identified weak understanding of the order of operations as one factor contributing to low achievement in mathematics. Local evidence also supports the need for PEMDAS-based interventions, as Philippine classroom observations show that guided practice, visual aids, and contextual word problems help improve comprehension, engagement, and confidence in solving multi-step problems (Magallanes & Ubalde, 2023). Within Paratong Integrated School, the Beginning of School Year Rapid Mathematics Assessment revealed varying levels of mathematical proficiency among 55 Grade 6 learners. Some learners were classified as Emerging - Not Proficient, Emerging - Low Proficient, and Developing - Nearly Proficient, while others reached Transitioning - Proficient and Highly Proficient levels. This profile indicates the need for differentiated, visual, and play-based instructional support to strengthen PEMDAS mastery and mathematical problem-solving.

This study is anchored on theories of learning, cognitive development, and mathematics education. Constructivist Theory explains that learners actively construct knowledge through exploration, experimentation, and discovery (Bruner, 1991). In PEMDAS-based activities, learners organize operations, identify relationships among numbers, and develop conceptual and procedural understanding through active engagement. Research confirms that learners who actively engage with problems develop deeper understanding and stronger problem-solving skills than those who rely only on memorization (Hand et al., 2024). Vygotsky's Sociocultural Theory emphasizes social interaction, scaffolding, and the Zone of Proximal Development (ZPD) in learning (Vygotsky, 1978). In this study, teacher guidance, peer collaboration, modeling, and guided practice support learners as they gradually gain independence in applying PEMDAS correctly. Cognitive Load Theory states that learning improves when instruction manages the limitations of working memory (Sweller, 1988). PEMDAS-based instructional materials reduce cognitive load by breaking multi-step problems into manageable steps, using visual cues, and guiding learners through the correct sequence of operations. Bruner's Spiral Curriculum Theory suggests that

concepts should be revisited at increasing levels of complexity (Bruner, 1960). The PEMDAS-based material follows this principle by beginning with basic operations, progressing to combined operations, and applying these to contextualized word problems. Behaviorist Theory emphasizes reinforcement, repetition, and immediate feedback in skill acquisition (Skinner, 1953). The Snake and Ladder Anchored to the Wizard activity provides repeated practice, correction of errors, and reinforcement of correct problem-solving procedures. Research indicates that structured practice using guided problem-solving materials improves accuracy and automaticity in mathematical skills (Sénéchal et al., 2024). Conceptually, the study follows an input-process-output framework. The input includes Grade 6 learners' prior knowledge based on RMA-BoSY results, pretest assessment, and the instructional material. The process involves intervention using PEMDAS-based word problems, visual aids, teacher scaffolding, peer collaboration, and posttest administration. The output is the developed Snake and Ladder Anchored to the Wizard instructional material and the expected improvement in learners' mathematical performance.

Although prior studies confirm the value of structured, scaffolded, and visual instructional materials in mathematics, PEMDAS-based materials remain underutilized in many Philippine classrooms. Large class sizes, limited teaching resources, and reliance on rote methods restrict the adoption of guided and visual approaches. Most local studies have focused on high-stakes testing or general mathematics achievement, leaving a gap in understanding how step-by-step, PEMDAS-focused instructional materials specifically affect Grade 6 learners' problem-solving skills (DepEd, 2022; Magallanes & Ubalde, 2023). This gap is important because Grade 6 is a critical stage for consolidating mathematical skills before learners proceed to higher-level mathematics. There is a need for empirical evidence on how localized, play-based, CPA-supported instructional materials can improve learners' ability to solve simple word problems using PEMDAS. Thus, this study develops and evaluates the Snake and Ladder Anchored to the Wizard instructional material as an interactive intervention for improving mathematical accuracy, conceptual understanding, procedural fluency, and learner confidence.

This study aligns primarily with SDG 4 (Quality Education) because it promotes inclusive, equitable, and effective mathematics instruction through guided, interactive, and low-cost learning materials. By improving foundational numeracy and problem-solving skills, the study supports better learning outcomes among Grade 6 learners. The study also supports SDG 5 (Gender Equality) by providing equal learning opportunities for all learners regardless of background through hands-on, scaffolded, and participatory problem-solving activities (UNESCO, 2020). In addition, it contributes to SDG 9 (Industry, Innovation, and Infrastructure) by demonstrating how simple and innovative instructional tools can improve learning outcomes in resource-constrained classrooms.

The study contributes to educational sustainability by strengthening foundational numeracy skills, supporting teachers with practical instructional strategies, and promoting learner-centered mathematics instruction. It also supports institutional sustainability by offering a low-cost, adaptable instructional material that schools can integrate into mathematics lessons, remediation activities, and localized curriculum support. From a socio-economic perspective, improved mathematical skills prepare learners for higher-level academic tasks and practical life situations involving computation, reasoning, and decision-making. The study also contributes to innovation and instructional sustainability by using play-based learning, CPA strategies, visual aids, and scaffolded word problems to address persistent mathematics difficulties without requiring expensive technology or advanced infrastructure. Overall, the research supports sustainable improvement in mathematics education by providing a contextualized, developmentally appropriate, and evidence-based instructional material that can benefit learners, teachers, school administrators, curriculum developers, parents, and future researchers.

2. Material and methods

2.1 Research Design

The study employed a quasi-experimental research design, specifically the pre-test–post-test one-group design, to determine the effectiveness of the developed PEMDAS-based instructional materials in improving the mathematical performance of Grade 6 learners. This design was appropriate because it allowed the researcher to compare learners' performance before and after the intervention in a natural classroom setting where full experimental control was limited. A quasi-experimental design is widely used in contemporary educational research, particularly when random assignment is not feasible due to classroom constraints but comparison between groups is still necessary (Cohen, Manion, & Morrison, 2022; Creswell & Creswell, 2021). The pre-test established the learners' baseline competencies in applying PEMDAS, while the post-test measured their performance after using the instructional materials. Recent methodological literature affirms that the pre-test–post-test one-group design provides strong evidence of intervention effectiveness because it accounts for both initial equivalence and subsequent improvements (Fraenkel, Wallen, & Hyun, 2021; McMillan & Schumacher, 2022). This procedure also aligns with Gay, Mills, and Airasian's (2020) assertion that pre-test–post-test measures are essential in determining the effectiveness of instructional innovations.

2.2 Locale of the Study

The study was conducted at Paratong Integrated School in the District of Bangar during the School Year 2025–2026. The school was selected because it reflects the common challenges experienced in rural elementary schools in the Philippines, particularly learners' difficulty in performing multi-step mathematical operations and applying PEMDAS in word problems.

2.3 Respondents of the Study

The respondents of the study were Grade 6 learners of Paratong Integrated School who were officially enrolled during the School Year 2025–2026. The study involved a total of 55 learners, composed of 30 males and 25 females. These learners served as the source of data for determining the effectiveness of the PEMDAS-based instructional materials.

2.4 Sample and Sampling Procedure

Total enumeration was used in selecting the respondents. All 55 Grade 6 learners enrolled at Paratong Integrated School during the covered school year were included in the study. This procedure was appropriate because the population was manageable and directly relevant to the purpose of the intervention.

2.5 Research Instrument

The study used a researcher-made PEMDAS achievement test as the pre-test and post-test instrument. The test contained grade-level appropriate items aligned with the Mathematics curriculum and measured learners' understanding of the hierarchy of operations, including parentheses or brackets, exponents, multiplication and division, and addition and subtraction. It included computational problems and situational word problems to assess both procedural and conceptual understanding. The intervention also used PEMDAS learning modules and activity sheets composed of step-by-step guided exercises, visual organizers, problem strips, and practice worksheets. These materials emphasized identifying operations, solving multi-step problems, checking solutions, and applying PEMDAS in real-life scenarios. The instructional materials were used to reinforce systematic problem-solving and reduce common learner errors.

2.6 Data Gathering Procedure

The data-gathering process was conducted in three phases. In the pre-intervention phase, the researcher administered the pre-test to determine the learners' baseline performance in solving equations using the order of operations. Learners were also oriented on the use of learning modules and problem-solving strategies, while their common difficulties, such as ignoring parentheses and misordering operations, were initially observed. During the intervention phase, structured PEMDAS-focused sessions were implemented. Learners participated in guided practice, pair activities, and independent seatwork using the CPA approach. The teacher facilitated step-by-step solving demonstrations, modeled the correct order of operations, and conducted error-analysis discussions. The Snake and Ladder Anchored to the Wizard instructional game was incorporated to strengthen learners' procedural fluency in solving mathematical expressions using PEMDAS. The game was played in small groups using a modified board, a die, player tokens, and teacher-prepared PEMDAS problem cards. Learners advanced on the board by solving problems correctly, while incorrect answers led to movement penalties or missed turns. Special Wizard Squares provided bonus or challenge questions to further reinforce mastery. In the post-intervention phase, the researcher administered a parallel form of the PEMDAS achievement test. The pre-test and post-test results were then compared to identify learning gains and determine the effectiveness of the intervention.

2.7 Data Analysis Techniques

The study used mean and standard deviation to summarize learners' performance in the pre-test and post-test. The mean was used to determine the learners' average scores, while the standard deviation described the variation in their performance before and after the intervention. Learners' scores were interpreted using performance levels ranging from Very Low to Very High. A paired sample t-test was also used to determine whether there was a significant difference between the pre-test and post-test scores of the same group of learners. This statistical treatment was appropriate because the study compared two related measures from the same participants. All statistical analyses were interpreted at the 0.05 level of significance.

2.8 Ethical Considerations

The study observed ethical standards to protect the rights, welfare, and privacy of the learners. Permission was obtained from the Schools Division Office and the school head of Paratong Integrated School before data collection. Written informed consent was secured from parents or legal guardians, and age-appropriate assent was obtained from the learners. Participants were informed about the purpose, procedures, voluntary nature of participation, and their right to withdraw at any time without penalty. Confidentiality was maintained by using codes or pseudonyms in test papers and data files. All collected data, including pre-test and post-test scores, worksheets, and observational notes, were used only for research purposes and securely stored. The study ensured that no learner was harmed or disadvantaged, and all participants were treated fairly and respectfully regardless of academic ability.

3. Results and Discussion

3.1 Mathematical Skills of Grade 6 Learners Using the CPA-Based PEMDAS Instructional Material

3.1.1 Mathematical Skills Before the Intervention

Before the use of the CPA approach through Snake and Ladder Anchored to the Wizard, the Grade 6 learners obtained a mean score of 8.89, interpreted as Low. This indicates that the learners had only basic mathematical understanding and had not yet achieved sufficient mastery in solving PEMDAS-based word problems. Many learners were classified under the Low and Very Low levels, while only a few reached the High level, showing that advanced mathematical proficiency was limited among the group. These results suggest that many learners experienced difficulty in conceptual understanding and problem-solving before the intervention. Since mathematical ideas are often abstract, learners may struggle when instruction does not provide concrete and visual support. This finding supports Fyfe et al. (2020), who reported that learners show lower conceptual understanding when instruction lacks concrete and visual representations. Similarly, Reinholz et al. (2021) found that traditional abstract teaching approaches often result in gaps in mathematical comprehension among elementary learners.

3.1.2 Mathematical Skills After the Intervention

After the implementation of the CPA approach through Snake and Ladder Anchored to the Wizard, the learners obtained a mean score of 15.02, interpreted as High. This shows a clear improvement from their pretest performance. Most learners who were previously classified under Low and Very Low improved to Moderate and High levels, while several learners achieved Very High scores, indicating better mastery of mathematical concepts and improved problem-solving performance. The improvement suggests that the CPA approach helped learners understand mathematics more effectively by moving from concrete materials to pictorial representations and then to abstract symbols. This result supports the findings of Hurst and Hurrell (2021), who reported that the CPA approach through Snake and Ladder Anchored to the Wizard significantly improves learners' conceptual understanding and problem-solving skills in mathematics. Similarly, Hidayati et al. (2022) found that learners exposed to CPA-based instruction achieved higher posttest scores compared to those taught using traditional methods. The findings also align with Aini et al. (2023), who emphasized that manipulatives and visual models enhance learners' engagement and retention of mathematical concepts.

3.1.3 Significant Difference Between Pretest and Posttest Scores

The paired sample t-test showed a computed t-value of -14.64 with a p-value of <0.001 , which is lower than the 0.05 level of significance. This indicates a statistically significant difference between the pretest and posttest scores of the learners. Therefore, the improvement in mathematical performance after the implementation of the CPA approach through Snake and Ladder Anchored to the Wizard was not due to chance but was significantly influenced by the intervention. This result implies that the CPA approach significantly improved learners' mathematical skills by providing hands-on learning, visual representation, and a gradual transition to abstract thinking. Consequently, the null hypothesis stating that there is no significant difference in the mathematical skills of Grade 6 learners before and after the use of the CPA approach is rejected, while the alternative hypothesis is accepted. This finding supports Sari et al. (2024), who reported that CPA-based instruction leads to statistically significant improvements in learners' mathematics performance. Likewise, Widodo et al. (2025) emphasized that structured, multisensory approaches such as CPA significantly improve learners' achievement and mathematical reasoning skills.

3.2 Overall Effectiveness of the CPA-Based Play-Based Instructional Material

Overall, the results show that the use of the CPA approach in developing PEMDAS-based instructional materials was effective in improving the mathematical skills of Grade 6 learners. The increase from a Low pretest mean score of 8.89 to a High posttest mean score of 15.02 demonstrates that the intervention strengthened learners' understanding of mathematical concepts, procedural fluency, and ability to solve PEMDAS-based word problems. The findings indicate that incorporating CPA-based strategies in mathematics, particularly through play-based instructional materials such as Snake and Ladder Anchored to the Wizard, can enhance learners' engagement, confidence, and problem-solving ability. The structured progression from concrete learning experiences to visual representations and symbolic computation made abstract mathematical concepts more accessible and easier to understand. Thus, the intervention provided meaningful support for improving learners' mathematical performance and strengthening their mastery of the order of operations.

4. Conclusion & Recommendations

The study concluded that the Grade 6 learners had low mathematical performance before the implementation of the CPA approach through the Snake and Ladder Anchored to the Wizard, indicating insufficient mastery of PEMDAS and difficulty in solving multi-step word problems. After the implementation of the CPA-based instructional material, learners' performance improved to a high level, showing better understanding of the order of operations, improved accuracy, and stronger problem-solving skills. The significant difference between the pretest and posttest scores confirmed that the CPA approach, combined with structured PEMDAS-based instructional materials, was effective in enhancing the mathematical

skills of Grade 6 learners.

Teachers are encouraged to integrate CPA-based strategies and PEMDAS-focused activities in mathematics instruction to strengthen learners' conceptual understanding and problem-solving skills. School administrators may support the development and use of localized instructional materials such as Snake and Ladder Anchored to the Wizard and provide teacher training on CPA implementation. Curriculum developers may incorporate structured, step-by-step, and contextualized PEMDAS-based word problem activities, while learners and parents may use guided practice materials to reinforce mathematical operations at school and at home. Future researchers may examine the long-term effects of CPA-based instructional materials, include comparison groups, or apply the intervention to other grade levels and mathematics topics.

The study was limited to the use of PEMDAS-based instructional materials for enhancing the mathematical skills of Grade 6 learners at Paratong Integrated School during School Year 2025–2026. It focused only on mathematical operations, multi-step word problems, and the correct application of PEMDAS using a quasi-experimental one-group pretest–posttest design. It did not cover advanced mathematics topics such as fractions, decimals, geometry, or algebra, and it did not assess the long-term retention of learners' mathematical skills beyond the intervention period.

Compliance with ethical standards

This study was conducted in compliance with institutional ethical standards, with permission secured from the Schools Division Office and the school head of Paratong Integrated School before data collection. Participation of the Grade 6 learners was voluntary, with informed consent obtained from parents or legal guardians and age-appropriate assent secured from the learners. Confidentiality was maintained through the use of codes or pseudonyms, and all data were used solely for research purposes. The author declares no conflict of interest, acknowledges the contributions of school officials, teachers, learners, parents, adviser, and panel members, and affirms that the manuscript is an original work, with copyright and authorship responsibility retained by the researcher.

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